Analysis of Semiconductor Microcavity Lasers Using Rate Equations

Gunnar Björk and Yoshihisa Yamamoto, *Member, IEEE*

***Abstract*—The rate equations for a microcavity semiconductor laser are solved and the steady-state behavior of the laser and some of its dynamic characteristics are investigated. It is shown that by manipulating the mode density and the spontaneous decay rates of the cavity modes, the threshold gain can be decreased and the modulation speed can be improved. However, in order to fully exploit the possibilities which the modification of the spontaneous decay opens up, the active material volume in the cavity must be smaller than a certain value. Subjects covered in the paper are threshold current using different definitions, population inversion factor, *L–I* curves, linewidth, and modulation response.**

I. Introduction

Modification of the spontaneous emission rate of an excited atom or an electron-hole pair opens up new possibilities in optical engineering. Both in the microwave region [1]–[4] and in the optical region [5]–[7], alteration of the spontaneous emission rate, and related phenomena, such as Rabi oscillation, have been demonstrated long ago. In the last few years several groups have been studying the effect of spontaneous emission enhancement/suppression in semiconductor material devices [7]–[10].

A rate equation analysis of an ideal microcavity laser with perfect population inversion, predicts that the threshold pump rate may be several orders of magnitude lower than that of a conventional laser [8]. The reason for this is that in an ideal microcavity laser the power dissipation of the active material is dominated by photon emission into one of the cavity modes. However, if the inverted medium is made from a semiconductor material, it is difficult to accomplish the perfect inversion assumed in an ideal laser. Furthermore, in order to fulfill the Bernard–Duraffourg condition [11], one needs a certain density of electrons and holes in the conduction and valence band, respectively. Some people have claimed that the need to fulfill this condition will seriously impair the potential for low threshold currents in semiconductors lasers. Others have said that in reducing the size of the laser, and thus the active material volume, the threshold current will go down, this being a pure size effect that has nothing to do with the alteration of the spontaneous emission rate. Others yet, have claimed that the threshold reduction can only be explained by cavity QED. The first purpose of this paper has been to clarify these issues. It will be shown that the transparency free-carrier density will set a lower limit to the threshold current, but only if the active material volume is "large." In Section III we will state a condition when the transparency free-carrier density does limit the threshold current reduction. This condition will define the term "large." We will also show that as long as the active material is "large" the decrease in threshold current with volume can be described as a pure size effect. However, when the active volume is "small," the threshold current can be decreased without decreasing the size of the cavity. (In [5], increase of the spontaneous emission rate for a constant cavity length is discussed.) This can be viewed as an cavity QED effect because it comes purely from the field–atom interaction modification by the cavity.

The second purpose of this paper is to show what to expect from a typical microcavity laser. We have calculated, e.g., *L–I* curves, linewidths, and modulation responses for both ideal and nonideal devices. It is shown that present devices operate far from the ideal limits mainly because of nonradiative recombination processes

II. Rate Equations

We describe the free-carrier density N in the active medium, and the photon population p in the cavity with rate equations. In a single-mode laser this is justified as long as the dephasing time of the active material dipole moment is much shorter than both the cavity photon lifetime τ_p and the spontaneous emission lifetime τ_{sp}. If this is the case, the dipole moment coupling between the inverted medium and the photons can be eliminated adiabatically [12]. We will come back to the practical constraints the lifetime condition puts on a microcavity laser. Assuming that the dipole moments can be eliminated adiabatically, the rate equations can be written

$$\frac{d}{dt}N = \frac{I}{qV} - \left(\frac{1-\beta}{\tau_{sp}} + \frac{\beta}{\tau_{sp}}\right)N - \frac{N}{\tau_{nr}} - \frac{gp}{V} \quad (1)$$

$$\frac{d}{dt}p = -(\gamma - g)p + \frac{\beta NV}{\tau_{sp}}. \quad (2)$$

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G. Björk is with the Department of Microwave Engineering, Royal Institute of Technology, S-100 44 Stockholm, Sweden.

Y. Yamamoto is with NTT Physical Science Research Laboratory, Musashino-shi, Tokyo 180, Japan.

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Here, I is the injection current, q is the electron charge, V is the volume of the *active* material (and this can be substantially smaller than the cavity volume), τ_{nr} is the nonradiative recombination lifetime, g is the active material gain when it sits in the cavity in s^{-1} and $\gamma \equiv 1/\tau_p$ is the cavity decay rate in s^{-1}. The spontaneous emission lifetime is defined as

$$\tau_{sp} \equiv 1 \Big/ \sum_i A_i \tag{3}$$

where A_i is the spontaneous emission rate of the active material into mode i. Note that the different rates A_i may differ due to the presence of a cavity. If the cavity bandwidth of a mode is larger than the gain bandwidth, the atoms may see a modified vacuum-field intensity in this mode (or rather quasi-mode), and the cavity decay rate into the mode will be modified. If the free spectral range of the cavity is smaller than the gain bandwidth, so there are many cavity modes within the gain bandwidth, the decay rate will be equal for all modes. In the former case, the rate A_i, as compared with the rate if the cavity was not present, can be suppressed by a factor $1 - R$ or enhanced with a factor $1/(1 - R)$ where R is the reflectivity of the cavity mirrors [5], [10], depending on whether the atomic transition frequency coincides with the cavity resonance frequency or not. The spontaneous emission coupling ratio β is defined as

$$\beta \equiv A_0 \Big/ \sum_i A_i \tag{4}$$

where index 0 indicates the optical mode which will eventually lase.

To describe the relation between the optical gain and the free-carrier density in the semiconductor we have assumed a linear gain model

$$g = g'(N - N_0) \tag{5}$$

where N_0 is the transparency carrier concentration of the gain material. This model, chosen for simplicity, should bring out at least the qualitative behavior of free-carrier concentration on the gain, and around $N = N_0$ it should give a good quantitative agreement.

From Einstein's relation between the A and B coefficients it is clear that for every mode, the spontaneous emission equals the stimulated emission when the average photon number in the mode is unity. Inspecting (2), using (5) we get

$$g' = \beta V/\tau_{sp}. \tag{6}$$

From (1), (2), (5), and (6) we can calculate some of the static and dynamic properties of the microcavity semiconductor laser. This is the topic of the subsequent sections. Before doing so, we will briefly discuss spontaneous emission rate enhancement. As can be seen from (4), there are several ways of increasing the spontaneous emission coupling ratio. The first is simply to reduce the number of modes that couples to the gain medium. The

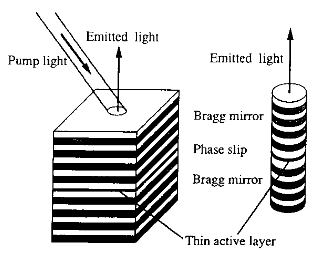

Fig. 1. Schematic example of one-dimensional (left) and three-dimensional (right) microcavities. The active layer is usually thin and the Bragg mirrors passive, but this need not be the case.

mode density per unit frequency can be written

$$\rho \approx \frac{8\pi V_c n^2 n_g}{\lambda_0^3 \nu} \tag{7}$$

where V_c is the cavity volume, n is the refractive index of the cavity material, n_g is the group index of the modes with an optical frequency $\approx \nu$, and λ_0 is the vacuum wavelength of these modes. If all modes have the same decay rate, the spontaneous emission coupling ratio will be roughly inversely proportional to the cavity volume. For a fixed number of interacting modes, β will increase if the spontaneous emission rates into other modes are suppressed, or the spontaneous emission rate into the lasing mode is enhanced. The enhancement/suppression is a true cavity QED effect and requires, as explained above, that the gain bandwidth is at least smaller than the cavity free spectral range. Depending on the balance between the three processes described, the spontaneous lifetime may either decrease or increase. In a planar (one-dimensional) dielectric cavity structure (Fig. 1), τ_{sp} will generally increase slightly when β is increased [9]. On the other hand, in a well designed, laterally guiding (three-dimensional) microcavity τ_{sp} can be reduced substantially when β is increased [10]. In our rate equations it is understood that τ_{sp} is the spontaneous lifetime of the active material when it sits in the cavity. The cavity spontaneous emission lifetime τ_{sp} may therefore differ substantially from that of the bulk active material.

III. Threshold Behavior

One of the most prominent and perhaps most promising features of the microcavity lasers is their potential of very low threshold current operation. In the first report to point this out, the authors dubbed them "virtually zero threshold lasers" [13]. While the devices certainly can have low threshold currents, it may be in the sub μA domain, it is certainly finite and, in general, well defined as will be shown. In semiconductor microcavity lasers, however, the picture is somewhat more complicated due to the fact that to invert the medium, the free-carrier density has to exceed a certain level. The question has been raised if this will prohibit very low threshold semiconductor lasers. As will be shown in this section, the answer to the question lies partly in how large the laser active volume is, and partly in how the threshold current is defined. We will look at three different threshold definitions and compare the resulting threshold currents.

The first and most widely used definition for the threshold current is that the net stimulated gain should equal the loss. Pumping harder, the net stimulated emission will rapidly become the dominant emission source and lasing will occur. Inspecting (2) we find that this is equivalent to

$$\gamma = \frac{\beta V}{\tau_{sp}} (N_{th1} - N_0). \tag{8}$$

From (2) it can be seen that the net stimulated steady-state gain can only equal the loss at an infinite photon number. Thus, strictly speaking, (8) is unphysical. However, in applying (8) it is implicitly assumed that when calculating the free-carrier density, and thus the gain, stimulated recombination can be neglected. Doing so we get

$$N \approx \frac{I}{qV(1/\tau_{sp} + 1/\tau_{nr})} \tag{9}$$

so that

$$N_{th1} = N_0 + \frac{\tau_{sp}\gamma}{\beta V} = N_0\left(1 + \frac{1}{\xi}\right) \tag{10}$$

where ξ is a dimensionless parameter defined by

$$\xi = \frac{N_0 \beta V}{\gamma \tau_{sp}}. \tag{11}$$

Note that ξ can be interpreted as the photon number in the lasing mode when $N = N_0$, that is, when the active medium is transparent. At this inversion level there is no net stimulated emission, and ξ is the ratio of spontaneous photon emission into the lasing mode $N_0 V\beta/\tau_{sp}$ and the cavity loss rate γ. As will be shown, if this number is larger than unity we need only to increase the free-carrier density slightly above the transparency value to get substantial stimulated emission. In this case the threshold is mainly determined by the material properties. If, on the other hand, ξ is much smaller than unity, we need to raise the free-carrier density substantially to offset the cavity loss, so the threshold will be mainly determined by the cavity properties.

Before calculating the threshold current, the population inversion factor will be derived. Using the definition $n_{sp} \equiv N/(N - N_0)$, and (10), the population inversion factor when (8) is fulfilled can be expressed

$$n_{sp,th1} = 1 + \xi. \tag{12}$$

As the photon number implicitly has been assumed to be infinity when (8) is fulfilled, this is also the population inversion factor of a microcavity laser well above threshold. In Fig. 2 the function has been drawn using dashed lines. It is clear that the active volume should be smaller than $\gamma\tau_{sp}/N_0\beta$ to assure a low population inversion parameter at and above threshold.

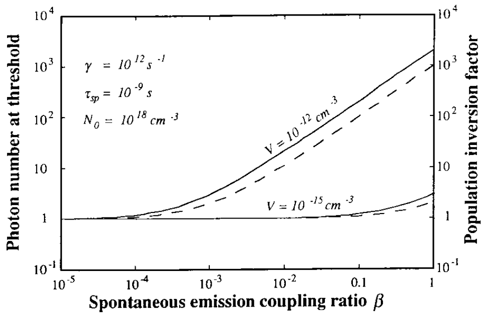

Fig. 2. The population inversion parameter at threshold (right vertical axis) versus the spontaneous emission coupling ratio, for two different active volumes. The nonradiative recombination rate has been assumed to be negligible compared to the radiative recombination rate. The dashed line is for the threshold definition (8), the solid line is for the definition $p_{th} = n_{sp,th}$. The solid line also shows the photon number at threshold (left vertical axis).

Using (9) and (10) we find that the threshold pump current will be

$$I_{th1} = \frac{q\gamma}{\beta}(1 + \xi)\left(1 + \frac{\tau_{sp}}{\tau_{nr}}\right). \tag{13}$$

Inspecting this equation one can conclude that irrespective of the value of ξ, there will always be a threshold current penalty as soon as τ_{nr} becomes smaller than τ_{sp}. This is expected, and it is presently a major obstacle to overcome in the device fabrication process, as the reported τ_{nr} values of laterally guided (three-dimensional) semiconductor microcavity lasers [14] are at least an order of magnitude smaller than the corresponding τ_{sp}. If we assume that we have an ideal active material so that we can neglect the nonradiative recombination, the expression for the threshold current will consist of two terms. The left term depends mainly on the properties of the cavity (β and γ), and the right term (using the definition of ξ and factoring out β and γ) depends mainly on the properties of the gain medium (N_0, V, and τ_{sp}). If ξ becomes sufficiently small (13) reduces to

$$I_{th1} \approx \frac{q\gamma}{\beta}\left(1 + \frac{\tau_{sp}}{\tau_{nr}}\right). \tag{14}$$

On the other hand, if $\xi >> 1$, the threshold current can be approximated by

$$I_{th1} \approx \frac{qN_0 V}{\tau_{sp}}\left(1 + \frac{\tau_{sp}}{\tau_{nr}}\right). \tag{15}$$

If we reduce ξ by reducing the volume of the active medium, the threshold current will decrease until the left term in (13) dominates. This threshold reduction is mainly a classical effect. We make the number of atoms smaller and thereby make the injection current to sustain the spontaneous emission into all nonlasing modes negligible compared to the lasing mode losses. Equation (14) is proportional to $1/\beta$ and sets the fundamental limit for the threshold gain for a given cavity. If nonradiative recombination is negligible, the ultimate threshold current is independent of both τ_{sp}, N_0, and V. It should be pointed out that in a conventional semiconductor laser in which the active material fills a substantial part of the cavity volume, β may be in the order of 10^{-5} to 10^{-4}, so the potential threshold current reduction is four to five orders of magnitude if we can just keep ξ small as β is increased. Reduction of the threshold current by increasing β can be considered mixture of a classical effect and cavity QED effect. If β simply reduces inversely with the cavity volume, as it will in a macroscopic or an ill designed microcavity laser, the reduction can still be viewed as an classical effect. The number of modes interacting with the active material has simply been reduced. If β can be increased faster than the inverse of the cavity volume, or without changing the cavity volume at all, it is a cavity QED effect. The presence of the cavity has selectively increased/decreased the coupling between the atoms and the vacuum modes. A plot of the threshold current (13) as a function of β is shown by the dashed dotted lines in Fig. 3 for typical microcavity semiconductor laser parameters. It can be seen in the figure that as long as $\beta < 10^{-3}$ so that $\xi < 1$ the threshold current is approximately given by (14) for the laser with the larger active volume. When $\beta > 10^{-3}$, the threshold current is roughly given by (15). The laser with the smaller active volume follows (14) as long as $\beta < 0.1$.

We note that if we have an ideal microcavity laser, with no nonradiative recombination, $\xi \approx 0$ and $\beta = 1$, then (14) simplifies to

$$I_{th1} = q\gamma. \tag{16}$$

This equation has a simple interpretation. In such an ideal laser the only loss mechanism is photons emitted into the lasing mode. The photon emission rate is exactly the rate at which we must inject new carriers in order to compensate for the loss. Since the photon loss rate in this case is γp, (16) tells us that the mean photon number at threshold is unity for a truly ideal laser!

Noting that the conventional definition of the threshold (8) strictly speaking is unphysical, in that the stimulated emission can never quite compensate for the optical loss, a better definition of the threshold would perhaps be that the net stimulated emission (that is, the stimulated emission minus the stimulated absorbtion) shall equal the spontaneous emission. The reason this (in our opinion, better) definition is seldom used, is that the definition involves the photon number and not simply the gain. Thus, a nonlinear analysis must be undertaken. As will be shown below, the two definitions will give the same threshold current to within a factor of two, the new definition giving the lower estimate. Looking at (2), (5), and (6) at steady state, one easily finds that the new definition leads to a threshold photon number given by

$$p_{th2} \equiv \frac{N_{th}}{N_{th} - N_0} = n_{sp.th2}. \tag{17}$$

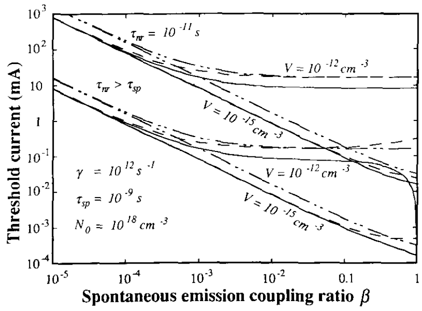

Fig. 3. Threshold current versus spontaneous emission coupling ratio. Dashed-dotted lines correspond to the threshold definition that the net stimulated gain equals the loss, dashed lines correspond to $p_{th} \equiv n_{sp,th}$, and solid lines correspond to the definition $p_{th} \equiv 1$.

Here, N_{th2} is the free-carrier concentration at threshold and $n_{sp,th2}$ is the population inversion parameter at this threshold. Inserting this equation in (2) one can express the threshold photon number in the rate equation parameters as

$$p_{th2} = 1 + \frac{2N_0\beta V}{\gamma\tau_{sp}} = 1 + 2\xi. \tag{18}$$

This is always a finite number. Per definition, it is also the population inversion parameter at the new threshold. The function is drawn in Fig. 2 using solid lines. Solving (2) for the free-carrier concentration at threshold, one arrives at the equation

$$N_{th2} = \frac{N_0}{2}\left(2 + \frac{1}{\xi}\right). \tag{19}$$

From this equation it can be seen that with the threshold definition (17) the free-carrier density at threshold will always be larger than the transparency density N_0. Plugging (18) and (19) into (1) one finally finds the threshold current to be

$$\begin{aligned} I_{th2} &= \frac{q\gamma n_{sp,th2}}{2\beta}\left(1 + \beta\frac{\tau_{sp}}{\tau_{nr}}\right) \\ &= \frac{q\gamma}{2\beta}\left[(1 + 2\xi)\left(1 + \beta + \frac{\tau_{sp}}{\tau_{nr}}\right)\right]. \end{aligned} \tag{20}$$

In Fig. 3 the threshold current according to the definition in (17) is plotted using dashed lines. The equation will give roughly the same result as (13) when $\xi >> 1$ and it will be a factor of two smaller than (13) when $\xi << 1$.

A third threshold definition used is that the stimulated emission rate shall simply equal the spontaneous emission rate at threshold. The rationale behind this definition is that at this pump level, half of the photons emitted into the mode will be emitted coherently, the other half will be added noncoherently. Pumping harder, both the coherence properties and the quantum efficiency will improve rapidly due to the rapidly increasing stimulated emission. From the Einstein relation already mentioned, this definition of the threshold will imply that the mean photon number in the mode at threshold is unity:

$$p_{\mathrm{th3}} \equiv 1. \tag{21}$$

Using this definition in (2) at steady state, we immediately get the free-carrier density at threshold

$$N_{\mathrm{th3}} = \left(N_0 + \frac{\tau_{\mathrm{sp}}\gamma}{\beta V}\right)\Big/2 = \frac{N_0}{2}\left(1 + \frac{1}{\xi}\right). \tag{22}$$

Somewhat surprisingly we find that using the definition (21) of p_{th}, when β and V are sufficiently large (so that ξ is much larger than unity), the threshold free-carrier concentration will be $N_{th} = N_0/2$. The active medium has higher stimulated absorbtion than emission, but due to the spontaneous emission, the mean photon number in the mode will still be unity. The laser will thus emit photons at a rate γ.

The population inversion factor at threshold, using (21) as the threshold definition can be expressed

$$n_{\mathrm{sp,th3}} = \frac{1 + \xi}{1 - \xi} \approx \begin{cases} 1 + 2\xi & \text{if } \xi \ll 1 \\ -1 & \text{if } \xi \gg 1 \end{cases}. \tag{23}$$

The fact that the population inversion factor is -1 when ξ is large leads us to the conclusion that this threshold definition may not be suitable when $\xi > 1$. Having this in mind, and using (1) and (21), we find that the threshold pump current using (21) will be

$$\begin{aligned} I_{\mathrm{th3}} &= \frac{q}{2}\left[\frac{\gamma}{\beta}\left(1 + \beta + \frac{\tau_{\mathrm{sp}}}{\tau_{\mathrm{nr}}}\right) + \frac{N_0 V}{\tau_{\mathrm{sp}}}\left(1 - \beta + \frac{\tau_{\mathrm{sp}}}{\tau_{\mathrm{nr}}}\right)\right] \\ &= \frac{q\gamma}{2\beta}\left[1 + \beta + \frac{\tau_{\mathrm{sp}}}{\tau_{\mathrm{nr}}} + \xi\left(1 - \beta + \frac{\tau_{\mathrm{sp}}}{\tau_{\mathrm{nr}}}\right)\right]. \end{aligned} \tag{24}$$

In Fig. 3 the threshold current according to the definition in (21) is plotted using solid lines. Equation (21) will also give roughly the same result as (13) when $\xi \gg 1$ and will be a factor of two smaller than (13) when $\xi \ll 1$. The only case where there is a substantial discrepancy between the two former threshold definitions and the last one is when both the active volume and the spontaneous emission coupling ratio are large, and the nonradiative recombination is negligible. The difference is easily explained by looking at the photon number in the mode at the threshold defined by (17), remembering that in this regime the overall quantum efficiency is roughly unity (Fig. 2). To increase the photon number by a factor of ten, one has to increase the pump current by a factor of ten. At $\beta = 1$ the threshold photon number for the two threshold definitions differs by roughly a factor of 2000, so the threshold currents differ with the same factor.

After computing the different threshold currents we can make some comments on them. We find that as long as ξ is small, they predict the same threshold current within a factor of two. When ξ is larger than unity they still agree well, except when β is very close to unity. Within this regime it is difficult to make a clear definition when lasing begins since the transition from LED to laser behavior rather smooth. We note that the threshold definitions 1 and 2 will make a much more conservative estimate of the threshold current in this regime.

We would like to conclude this section with two numerical examples, one for a conventional semiconductor laser and one for a microcavity laser. A conventional GaAs semiconductor laser has a cavity volume of approximately $1 \times 2 \times 300$ μm. The active material volume can be estimated to be one tenth of this volume. The transparency free-carrier density in GaAs is approximately 10^{18} cm^{-3}, the spontaneous emission lifetime is 3 ns, the refractive index is 3.5, and the FWHM of the optical gain Γ is around 20 ns at an emission wavelength of 800 nm [11]. The loss rate for a cleaved facet FP laser is roughly 40 cm^{-1} or 4×10^{11} s^{-1}. Assuming that the decay rate into every mode is the same, we can estimate the spontaneous emission coupling ratio using (7) to be $\beta \approx \lambda^3\lambda_0/8\pi V_c\Gamma$, where λ is the wavelength in the GaAs cavity and λ_0 is that in free space. We have also approximated the group index with the bulk refractive index of GaAs. Using the figures above we find that β is approximately 3×10^{-5} and that $\xi \approx 1.6$. This is reasonable, since the population inversion parameter (which can be expressed $1 + \xi$) for such structures is often reported to be in the range 2–3. Nonradiative recombination can usually be neglected in these lasers, using (20) we find the threshold current for the device to be 5.5 mA. This roughly the correct value. From the value of ξ we find that the threshold current could be reduced to merely half its calculated value by simply making the active volume smaller.

The next example we will calculate is a 5 μm diameter $In_{0.2}Ga_{0.8}As$ single quantum well laser emitting at 980 nm [14]. The QW is 80 Å thick and we have estimated the cavity length to be ten wavelengths. The cavity volume is thus around 65 μm^3 and the active volume is 0.15 μm^3. The Bragg mirror transmission was 0.1%, so the decay rate can be estimated to be 3.3×10^{10} s^{-1}. We assume that the gain linewidth, the spontaneous emission rate, and the transparency free-carrier density are the same as in the last example. Since the free spectral range of the cavity is roughly equal to the gain bandwidth, the reduction of the threshold current for this device is not a cavity QED effect, in spite of its small size. We can again assume that the decay rates for all modes are equal and we get $\beta \approx 3.6 \times 10^{-3}$ and $\xi \approx 5.4$. The nonradiative lifetime is stated to be shorter than 100 ps, we will assume the value 50 ps. From (20) the threshold current is found to be 0.2 mA. This is almost an order of magnitude smaller than the reported value of 1.5 mA. Due to the uncertainties in the estimated values of N_0, τ_{sp}, and τ_{nr} we cannot expect a perfect agreement between theory and experiment. What the example shows, however, is that again the threshold current probably could be reduced by making the active volume smaller even though cavity QED plays no role for this device.

IV. Steady-State Characteristics

A. Output Power

The output power characteristics of a microcavity laser is radically different from an ordinary laser. If the microcavity laser supports only one mode within the its gain bandwidth, and if nonradiative recombination is negligible, then photon emission is the only means of power dissipation and the quantum efficiency of the device must be unity both below and above threshold. This was noted long ago and one then expects a device with a smooth transition from LED operation to laser operation.

Using (1), (2), (5), and (6) one can express the photon number as a function of the pump current. However, for mathematical simplicity, it is easier to express the pump current as a function of the photon number. The result is

$$I = \frac{q\gamma}{\beta}\left[\frac{p}{1+p}(1+\xi)\left(1+\beta p+\frac{\tau_{sp}}{\tau_{nr}}\right)-\xi\beta p\right]. \quad (25)$$

In Fig. 4, input versus output curves are drawn for three different combination of parameters, and it can be seen that except when $\beta \approx 1$ there is a sharp increase in the photon number (and output power) as soon as the photon number exceeds unity. This is an argument for the threshold definition (21). When $\beta \approx 1$ the transition from below to above threshold is smooth, provided that the nonradiative recombination is negligible. It is thus difficult to make an unambiguous definition of the threshold pump current based on the input–output or the gain characteristics when $\beta \approx 1$. In drawing the output power axis we have assumed that there are no optical losses except mirror losses. If one wishes to include other optical losses, one can simply multiply the plotted output power with $(\gamma - \gamma_0)/\gamma$, where γ is still the total optical decay rate, and γ_0 is the total decay rate into all other modes except the lasing mode. Of course, when the injection current is many times its threshold value, (25) simplifies to

$$I = q\gamma p. \quad (26)$$

The fast stimulated emission decay rate combined with the assumption that mirror loss is the only optical decay process will assure the unity quantum efficiency manifested by (26). In the limit $\beta = 1$ and $\tau_{nr} >> \tau_{sp}$ one also arrives at (26) from (25) at all pump rates, as expected.

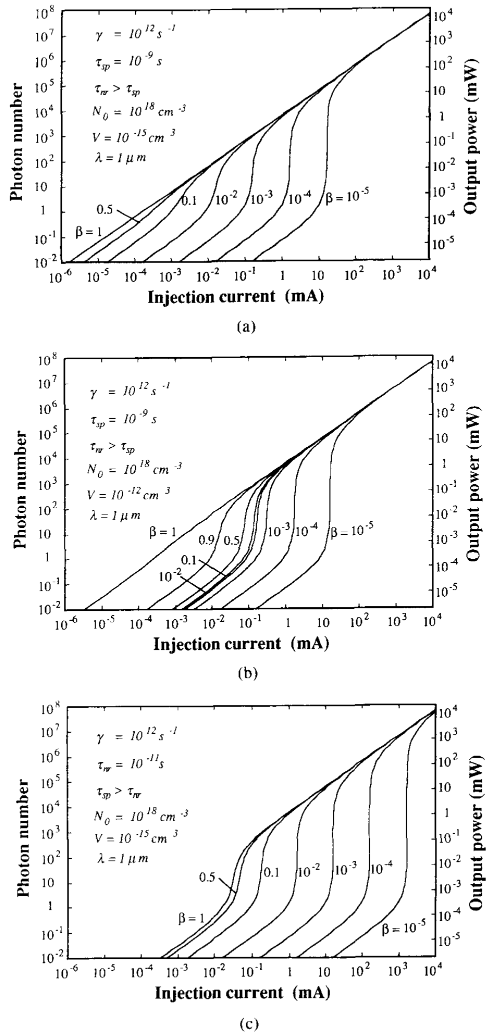

Fig. 4. The mean photon number in the mode (left axis) and the corresponding total output power ($\lambda = 1\ \mu m$, right axis) versus the pump current. In (a) and (c), ξ is always smaller or equal to unity, in (b) ξ is larger than unity when $\beta > 10^{-3}$. In (a) and (b) nonradiative recombination is negligible, but in (c) it dominates below threshold.

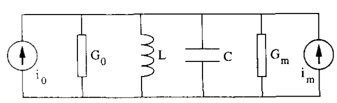

Fig. 5. Laser equivalent electrical circuit for linewidth calculation. The noise currents i_0 and i_m associated with the load conductance G_0 and the negative conductance G_m, respectively, are assumed to have flat spectra within the cold cavity bandwidth.

B. Linewidth

To get information about the frequency noise spectrum of microcavity lasers, one has to start with optical field equations and subsequently assign the correct noise sources. This is beyond the scope of this paper, but to at least get a feeling for the spectral purity of the microcavity semiconductor lasers we have calculated the linewidth, using an equivalent electrical circuit model (Fig. 5) described in [15]. This model neglects the gain-refractive index coupling of the active material and thus underestimates the linewidth above the threshold by a factor $1 + \alpha^2$, where α is the linewidth enhancement parameter. However, the model is simple to use and brings out the main features of the spectral linewidth of microcavity lasers. With this said, the reader is alerted that in order to properly include the effect of α, (31)–(33) should be multiplied by $1 + \alpha^2$.

$$\Delta\nu = \frac{\nu}{Q} = \frac{G_0}{2\pi C}\left(1 + \frac{G_m}{G_0}\right). \tag{27}$$

Here, Q is the total Q value of the cavity including all gain and loss mechanisms, G_0 is the circuit conductance due to the output coupling loss, C is the circuit capacitance, and G_m is the negative conductance representing the net stimulated emission. The noise currents associated with the load and the negative conductance are i_0 and i_m, respectively. The only assumption we have to do about these are that their spectral densities are constant over the cavity bandwidth. It is straight forward to identify the output coupling loss rate γ with the ratio G_0/C. The ratio between the photon loss and the net stimulated emission can be found inspecting (2) and the result for the linewidth is

$$\Delta\nu = \frac{1}{2\pi}\left(\gamma - \frac{\beta V}{\tau_{sp}}(N - N_0)\right). \tag{28}$$

Far below threshold where $N \approx 0$ (28) reduces to

$$\Delta\nu \approx \frac{1}{2\pi}\left(\gamma + \frac{\beta V N_0}{\tau_{sp}}\right). \tag{29}$$

This is the cold cavity bandwidth of a cavity with absorbing material inside, and it is wider than the cold cavity bandwidth of the empty cavity $\gamma/2\pi$. To get an expression for the other limit, above threshold, we observe that in steady state we have

$$\frac{\beta V}{\tau_{sp}}(N - N_0) = \gamma - \frac{\beta V N}{\tau_{sp} p} \tag{30}$$

from (2). Using this relation in (28) and noting that per definition $N = n_{sp}(N - N_0)$ one gets

$$\Delta\nu = \frac{\beta V n_{sp}(N - N_0)}{2\pi\tau_{sp} p} \approx \frac{\gamma n_{sp}}{2\pi p} \tag{31}$$

where in the last step we have used (30) and the fact that high above threshold the last term of the right-hand side of (30) vanishes. This equation is an equivalent formulation of the Schawlow–Townes linewidth formula. High above threshold we can replace n_{sp} using (12), and we get

$$\Delta\nu \approx \frac{\gamma(1 + \xi)}{2\pi p}. \tag{32}$$

In order to keep the linewidth low it is preferable to keep ξ below unity.

To write (31) in a more conventional way we introduce the total emitted optical power $P_e = h\nu p\gamma$ where h is Planck's constant. We also introduce the cold (and empty) cavity bandwidth $\Delta\nu_c = \gamma/2\pi$. One finds that (31) is equivalent to the expression

$$\Delta\nu = \frac{2\pi h\nu(\Delta\nu_c)^2 n_{sp}}{P_e}. \tag{33}$$

Equation (27) is known to give a correct result irrespective of the particular laser structure (Fabry–Perot, distributed Bragg reflector, etc.) if $\gamma_l L < 1$ where γ_l is the optical loss per unit cavity length, and L is the cavity length [16]. We can express γ_l in γ as

$$\gamma_l \approx \gamma/v_g \tag{34}$$

where v_g is the group velocity of the counter-propagating waves inside the cavity. Using the relation $v_g \approx \lambda\nu$, where λ is the wavelength in the cavity, we find that

$$\gamma_l L \approx \frac{\gamma}{\nu}\frac{L}{\lambda}. \tag{35}$$

Since, per definition in a microcavity laser, the last factor in (35) is of the order of unity, $\gamma_l L$ will be smaller than unity as long as the decay rate is smaller than the optical frequency. This is true even for moderately (50%) reflecting cavity mirrors which proves the validity of using (28) for microcavity lasers.

In Fig. 6, the linewidth of different microcavity semiconductor lasers are plotted versus the pump current using (28). It has been assumed that optical losses apart from mirror losses are negligible. From the curves we see that most microcavity configurations will come close to the ideal limit where $n_{sp} = 1$ and the overall quantum efficiency is unity ($P_e = Ih\nu/q$). The only exception, as foreseen in the last section, is when the active volume of the laser is larger than $\gamma\tau_{sp}/N_0\beta$ or when $\tau_{nr} > \tau_{sp}$. It is also seen that in general the linewidth narrows abruptly as soon as the threshold current is exceeded. When β is unity and nonradiative recombination is negligible, however, the transition is smooth and a distinct threshold is difficult to identify. This is in complete agreement with the results in the last section.

C. Cavity Constraints

As mentioned in the first paragraph of Section II, the rate equations (1) and (2) are only valid in this simple form if the dipole decay time τ_d is much shorter than τ_{sp} and τ_p. This is usually assumed to be the case, and we will now show that in a microcavity laser it is actually difficult to break this condition on τ_p.

The round-trip gain in any laser must roughly equal the round-trip loss. Thus, in a symmetric laser with mirror reflectivity R we have

$$R^2 \exp(2\gamma_l L) \approx 1. \tag{36}$$

From this equation we can express $\gamma_l L$ in the mirror transmittivity as

$$\gamma_l L \approx -\ln(R) \approx T. \tag{37}$$

Using (35) and assuming that L/λ is roughly unity one gets

$$T \approx \frac{\gamma}{\nu} \ll \frac{1}{\nu\tau_d}. \tag{38}$$

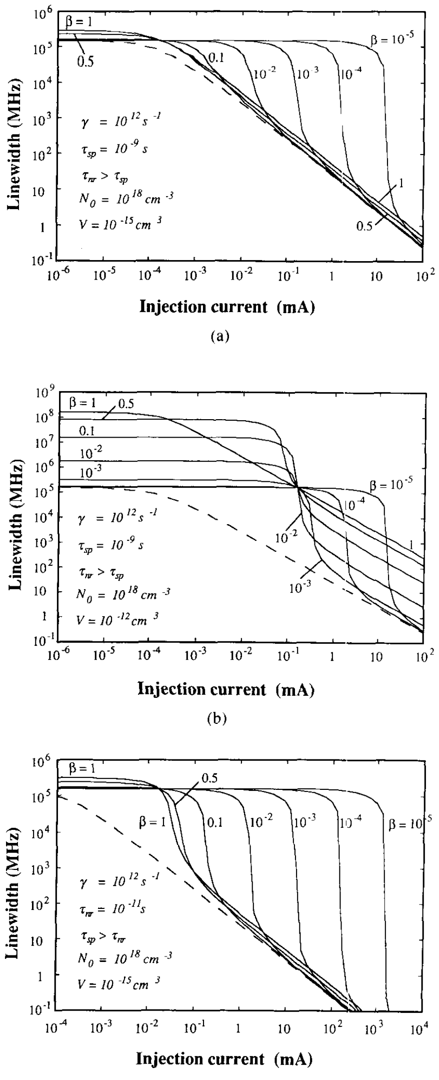

Fig. 6. The linewidth versus the pump current for different laser structures. The parameters in (a)-(c) correspond to those in Fig. 4(a)-(c), respectively. The linewidth enhancement parameter α has been assumed to be zero. The dashed line represents the ideal linewidth limit.

Putting typical numbers for a semiconductor material into (38), $\nu = 300$ THz, $\tau_d = 10^{-14}$ s, we find that T must be larger than 0.3 to break the above condition. To express the condition in terms of the cold (and empty) cavity Q_c value we use the relation $Q_c = 2\pi\nu/\gamma$ and get

$$Q_c < 2\pi\nu\tau_{sp}. \tag{39}$$

For the same numbers used above, the cavity Q must be less than roughly twenty before we expect the rate equations to lose validity.

V. Current Modulation Characteristics

The microcavity semiconductor lasers also promise to have a large modulation bandwidth, both below and above threshold. Below threshold, the modulation speed is limited by the spontaneous emission lifetime. As has already been discussed, one should be able to shorten the spontaneous emission lifetime substantially in a well designed and fabricated three-dimensional cavity. This immediately leads to a wider modulation bandwidth without any penalty in terms of higher threshold currents, as the threshold current is independent of τ_{sp} as long as ξ is small. However, since ξ is proportional to β, the requirement that ξ remains small will be tougher to accomplish since β in general increases if the spontaneous lifetime is increased. Above threshold, the modulation bandwidth is limited by either the stimulated decay rate, or the cavity decay rate, depending on which of them is the slowest process. Pumping sufficiently hard, the stimulated decay rate can always be made faster than the cavity decay rate.

The mode volume of a single transverse mode microcavity laser is roughly λ/L_c smaller than that of a conventional semiconductor laser, where L_c is the cavity length of the conventional laser. The active layer areas perpendicular to the injection current flow will have the same ratio. However, the threshold current may drop faster than this ratio, because the increase of β is not simply due to the fact that we have fewer modes in the cavity as the volume decrease. With a clever cavity design we can also selectively alter the decay rates of the different modes in a favorable way. Thus, at a given allowable current density per unit active area, one should be able to pump higher above threshold. Since the cavity area to volume ratio will increase as the cavity is made smaller, one should in principle be able to pump even harder, if the device dissipates heat into all three dimensions.

To find the small-signal injection current modulation response, we separate the injection current, the photon number and the free-carrier number into a steady-state term (indexed ss) and a fluctuating terms as $I = I_{ss} + \Delta I$, $p = p_{ss} + \Delta p$, and $N = N_{ss} + \Delta N$. Inserting these into the rate equations we get the steady-state solution already discussed, plus a new set of dynamic equations. Linearizing the dynamic equations and Fourier transforming one gets the intensity modulation response function

$$\frac{\Delta p(\Omega)}{\Delta I(\Omega)} = \frac{1/qV}{\dfrac{\beta}{\tau_{sp}}(N_{ss} - N_0) + \left(\dfrac{\beta p_{ss}}{\tau_{sp}} + \dfrac{1}{\tau_{sp}} + \dfrac{1}{\tau_{nr}} + j\Omega\right)\left(\dfrac{\gamma\tau_{sp}}{\beta V} - (N_{ss} - N_0) + j\dfrac{\Omega\tau_{sp}}{\beta V}\right)(1 + p_{ss})} \tag{40}$$

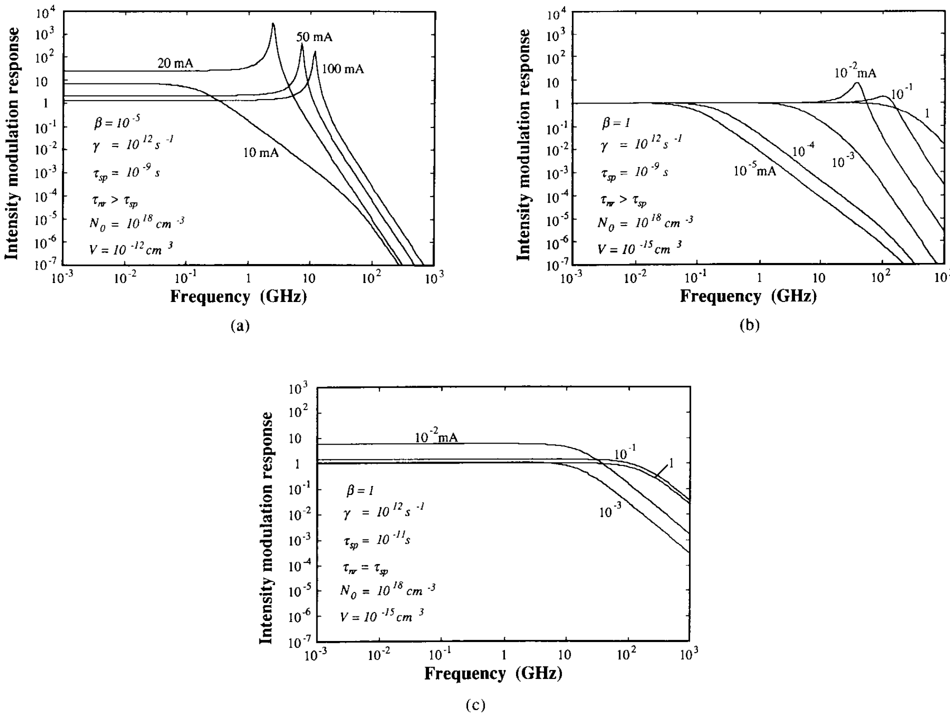

Fig. 7. The normalized intensity modulation response versus the modulation frequency for direct current modulation. In (a) the parameters are those of a conventional laser, in (b) the parameters are those of a $\beta = 1$ microcavity laser, and in (c) the parameters are those of a $\beta = 1$ microcavity laser with enhanced spontaneous decay rate.

where Ω is the angular frequency. Inserting the values for p_{ss}, and N_{ss} derived from Section III, the expression for the modulation response becomes messy. Therefore we don't give the expression for the modulation response as a function of the steady-state injection current and frequency, but restrict ourselves to plotting the absolute square of the normalized intensity modulation response as a function of frequency with I_{ss} as a parameter. The modulation response has been normalized by multiplying it by I_{ss}/p_{ss} so that the response function expresses the relative photon number modulation in the relative current modulation. Thus, if the normalized response function is unity, a 10% current modulation will result in a 10% photon number (or output power) modulation. In Fig. 7, we have plotted the absolute square of the normalized intensity response function for different sets of microcavity semiconductor laser parameters at different pump levels. All lasers behave essentially equal, below and near threshold the modulation response cuts off around the inverse of the spontaneous emission lifetime. Pumping harder and harder the modulation bandwidth will increase until its cutoff value reaches the inverse of the cavity lifetime. This is the ultimate limit. What is worth noticing is that for a laser with $\beta = 1$ and $\gamma = 10^{12}$ cm^{-1}, one reaches this ultimate bandwidth at an injection current of around 1 mA, less than a typcial threshold current for a conventional semiconductor laser!

VI. Beta Modulation Characteristics

An interesting alternative to modulating the intensity by modulating the current, is to modulate the intensity by varying the spontaneous emission coupling ratio β [17]. Above threshold nothing would be gained by β modulation; direct current modulation would be much simpler. Below threshold, however, one could get a very large modulation bandwidth *provided that one could change β without affecting the spontaneous emission lifetime.* As can be seen from (1), below threshold where the last term on the RHS is negligible, the population inversion is independent of β. Thus, by modulating the spatial distribution of the spontaneous emission by varying β, the modulation bandwidth would be proportional to the cavity decay rate. Following the same procedure as in the last section, one can derive the Fourier transform of $\Delta p(\Omega)$.

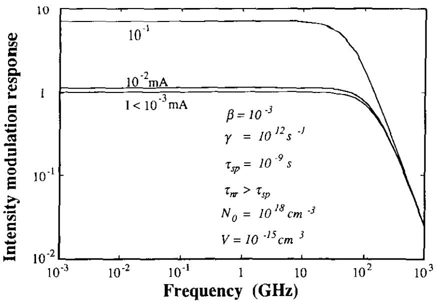

Fig. 8. The normalized intensity modulation response versus the modulation frequency for beta modulation. The high response function for $I = 0.1$ mA is due to the onset of oscillation, so a p versus β curve would have a kink at this value.

$$\frac{\Delta p(\Omega)}{\Delta \beta(\Omega)} = \frac{\dfrac{V}{\tau_{sp}}(N_{ss}(p_{ss}+1) - N_0 p_{ss}) - \dfrac{V\beta(N_{ss}-N_0)(p_{ss}+1)p_{ss}}{\tau_{sp}(1+\beta p_{ss}+\tau_{sp}/\tau_{nr}+j\Omega\tau_{sp})}}{\gamma - \dfrac{\beta V}{\tau_{sp}}(N_{ss}-N_0) + j\Omega + \dfrac{V\beta^2(N_{ss}-N_0)(p_{ss}+1)}{\tau_{sp}(1+\beta p_{ss}+\tau_{sp}/\tau_{nr}+j\Omega\tau_{sp})}}. \tag{41}$$

In Fig. 8, we have plotted the absolute square of the normalized beta modulation response function (the absolute square of (41) multiplied by β/p_{ss}). As long as ξ is small the bandwidth is roughly given by γ as can be seen from the denominator in (41).

In theory one could modulate β by shifting the wavelength of the emitted light out of the cavity resonance using, e.g., the quantum confined optical Stark-shift effect [18], [19]. In a three-dimensional cavity this would generally lead to modification of the spontaneous emission lifetime, but in a planar (one-dimensional) structure, the spontaneous lifetime varies much slower than β when the emission wavelength is tuned out of resonance with the cavity. Possibly, one could design a structure where only β varies and τ_{sp} remains constant. However, the reader should bear in mind that the requirement of constant spontaneous emission lifetime and the difficulties involved with electrical time constants associated with the Stark shifting will probably set the upper limit to the modulation speed substantially lower than γ for yet some time to come. The reader should also be warned that the analysis presented here does not take nonlinear gain phenomena into account. These may also limit the modulation bandwidth.

VII. Conclusion

The static and some of the dynamic characteristics of semiconductor microcavity lasers were investigated using rate equations. It was shown that in general, the different threshold conditions lead to roughly the same prediction for the threshold current. The only exception was when β is close to unity, nonradiative recombination is negligible, and the active volume is greater than $\tau_{sp}\gamma/N_0\beta$. It was also shown that only when the active volume is smaller than the above value is it possible to reduce the threshold by increasing β.

The linewidth high above threshold was shown to be equal to $\gamma(1 + \xi)/2\pi p$. Thus it is essential to keep ξ below unity to keep the linewidth near the ideal limit. In most cases there was seen to be distinct linewidth narrowing near the threshold point. When β is near unity, however, the narrowing was very slow which makes it difficult to define an unambigous threshold current.

The intensity modulation response was shown to be not much different from a conventional semiconductor laser. It is plausible that one may be able to pump microcavity lasers higher above threshold than conventional semiconductor lasers can be driven, so there is good hope that the modulation bandwidth of microcavity lasers can be made wider. Two possibilities to increase the modulation bandwidth when operating below threshold were outlined. One was to decrease the spontaneous emission lifetime by cavity QED effects, the other was to modulate β instead of the injection current.

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Gunnar Björk, photograph and biography not available at the time of publication.

Yoshihisa Yamamoto (S'75-M'81), photograph and biography not available at the time of publication.